Algorithms and Complexity
Solutions to Assignment 2

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November 9, 2000

1 Workshop problem

1.1 Terms

An instance of the Workshop problem is defined by n commands $o_1, \ldots, o_n$ of worth $e_1, \ldots, e_n$, a set of pairs (o_i, o_j) and a minimum target income K.

1.2 Idea

Show that IS $\leq_p$ Workshop (IS stands for Independent Set). Since IS $\in$ **NP-complete**, it would prove that Workshop $\in$ **NP-complete**.

1.3 Proof

Workshop $\in$ NP :

Given a solution $(o_i)_{i \in I}$ to Workshop, it can be checked in $O(n)$ that $\sum_{i \in I} e_i \geq K$.

Polynomial reduction from IS to Workshop :

An instance of the Workshop problem can be represented by a graph G in which the nodes are $o_1, \ldots, o_n$ and the edges are the pairs (o_i, o_j) and in which each node o_i has a weight e_i. Solving this instance of Workshop corresponds to finding an Independent Set $(o_i)_{i \in IS}$ over G so that

$$\sum_{i \in IS} e_i \geq K$$

When $e_1 = e_2 = \ldots = 1$, it appears that the solution of Workshop is also the solution of Independent Set for which an Independent Set of size K is searched.

Thus, we can define a straightforward reduction from IS to Workshop, which just consists in changing the graph representation of IS into an instance of Workshop in which $\forall i, e_i = 1$, and in which each incompatible pair (o_i, o_j) is represented by an edge. This can be done in polynomial time.

2 Find the mistake !

2.1 Terms

Given any graph $G = (V, E)$, VC_G a Vertex Cover of G, and IS_G an Independent Set of G, find the fault in:

$$\left\{ \begin{array}{l} \forall\ G, IS_G = V - VC_G \\ \text{Vertex Cover} \in \mathbf{APX} \end{array} \right. \quad \Rightarrow \quad \text{Independent Set} \in \mathbf{APX}$$

2.2 Idea

The assumption $\forall\ G, \quad IS_G = V - VC_G$ is false.

2.3 Proof

As an example of the contrary, let's G be a Clique of n nodes ($n > 2$). Thus, each node in G is connected with all the other nodes of G.
Then, if VC_G is a Vertex Cover of G containing p nodes ($p < n - 1$), then $I = V - VC_G$ contains at least 2 nodes and thus is not an Independent Set, since all nodes are interconnected. So,

$$IS_{Clique} \neq V - VC_{Clique}$$